

Electric-field Manipulation of the Landé g Tensor of Holes in $\text{In}_{0.5}\text{Ga}_{0.5}\text{As}/\text{GaAs}$ Self-assembled Quantum Dots

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The effect of an electric field on spin precession in $\text{In}_{0.5}\text{Ga}_{0.5}\text{As}/\text{GaAs}$ self-assembled quantum dots is calculated using multiband real-space envelope-function theory. The dependence of the Landé g tensor on electric fields should permit high-frequency g tensor modulation resonance, as well as direct, nonresonant electric-field control of the hole spin. Subharmonic resonances have also been found in g tensor modulation resonance of the holes, due to the strong quadratic dependence of components of the hole g tensor on the electric field.

I. INTRODUCTION

Since the initial proposals of spin-based quantum information processing^{1,2} in solids, methods of controlling individual spins in a scalable fashion have drawn considerable attention^{3–5}. An especially attractive method for independently controlling spins spaced much closer than an optical spot size is to locally modify some of the spin's properties with an electric field. This approach's advantages include well-developed and commercial techniques for generating large arrays of independently controllable electric gates. Initial proposals focused on modifying the resonant frequency of a spin in a magnetic field using spatially-dependent g factors² or hyperfine fields¹, however this approach requires a spatially-extended always-on microwave field with which local spins are brought in and out of resonance. A superior approach replaces electric-field control of a resonance frequency with electric-field control of the magnetic or pseudomagnetic field experienced by the spin, which can produce spin resonance when the electric field is oscillated at the resonance frequency. Electric-field control of the magnetic field has been demonstrated by moving an electron back and forth (using electric field control of the location of quantum confinement of a quantum dot) in an inhomogeneous magnetic field⁶. Electric-field control of a pseudomagnetic field has largely focused on modifying the orbital moment of an electron, which then affects the g tensor through the spin-orbit interaction; this approach goes by the name g tensor modulation resonance, and has been demonstrated in quantum wells⁷ and also proposed for electron spin manipulation in quantum dots⁸, for donor-bound electrons⁹, and for holes in quantum dot molecules¹⁰.

Advantages of using hole spins instead of electron spins for g tensor modulation resonance include the larger spin-orbit interaction and orbital angular momenta in the valence band, the stronger dependence (and asymmetry) of hole g tensors on structural shape^{11–13}, and possibly reduced hyperfine interactions. The larger spin-orbit interaction for holes and larger orbital angular momenta lead to larger hole g tensor components. If the fractional

change in orbital angular momentum with applied electric field is comparable for electrons and holes, then the variation in g tensor components should be much larger for holes, leading to much more rapid spin manipulation. To achieve rapid g tensor modulation resonance it is especially advantageous to have one tensor component that changes as the other is larger unaffected⁸. This can be achieved by considering a highly anisotropic dot shape. Zero in-plane g tensor components were predicted for holes in self-assembled InAs/GaAs quantum dots with a circular footprint^{12,14}, although once the footprint becomes elliptical these g tensor components grow rapidly¹². Other calculations have shown non-zero in-plane g tensor components in Ge/Si ,¹⁵ and III-V nanowhisker¹⁶ quantum dots as well as in a two-dot InAs/GaAs quantum dot molecule¹⁰. Experimental measurements have seen small, but non-zero in-plane hole g factors in InP/InGaP ¹⁷, CdSe/ZnSe ¹⁸ and $\text{InGaAs}/\text{GaAs}$ quantum dots¹⁹. As an extreme example, in a Ge/Si nanowire quantum dot, electrical control of the g tensor component parallel to the wire has been experimentally realized by varying the voltage on the electrostatic gates which define the dot²⁰. Another extreme asymmetric structure is the vertically-coupled quantum-dot molecule, in which efficient hole g tensor modulation by electric fields has been predicted¹⁰, including the possibility of spin echo²¹.

Hole spins have very short lifetimes in bulk, due to angular-momentum mixing and state degeneracy typically found in the valence band²², but this lifetime increases in quantum dots up to the same order of magnitude as electron spin lifetimes²³. Because the hole Bloch functions have p -like character, the contact term of the hyperfine coupling to the nuclear spin is zero, leaving only orbital hyperfine coupling to the nuclear spin. The resulting hyperfine interaction has been shown to be highly anisotropic for holes in quantum dots, with a pure heavy-hole state with the pseudospin aligned along the \hat{z} axis having little or no hyperfine coupling²⁴. Recent experiments have found hole T_2^* times of $\sim 100 - 490$ ns²⁵. In order to make a quantum computer, at least 10^4 operations must be performed during the decoherence time.²⁶

These recent experimental T_2 values suggest a minimum spin manipulation time no shorter than 10-50 ps.

Here we predict that both resonant and nonresonant spin manipulation is possible in holes in a single $\text{In}_{0.5}\text{Ga}_{0.5}\text{As}/\text{GaAs}$ quantum dot using a single vertical electrical gate, and that the spin manipulation times are more rapid than those for electron spins in the same quantum dots. We also find that the nonlinearity of the g tensor components with applied electric field is highly anisotropic, with a much stronger quadratic electric field dependence of the g tensor component parallel to the electric field direction than perpendicular to it. Such quadratic electric field dependences have previously been studied for systems of high symmetry, like donor states⁹, for which the linear electric field dependence vanishes. Here we find that these nonlinear g tensor components generate highly anisotropic *subharmonic* resonances which effectively manipulate the hole spin. For magnetic fields of $\sim 5 - 10$ T the spin manipulation times are of the order of 20-30 ps for electric fields $\sim 150 - 200$ kV/cm.

II. HOLE g TENSOR COMPONENTS IN A SINGLE QUANTUM DOT

A. Theoretical method

We have computed g tensors for holes confined in lens-shaped $\text{In}_{0.5}\text{Ga}_{0.5}\text{As}/\text{GaAs}$ self-assembled quantum dots as a function of an electric field applied along the growth direction and ranging from -150 kV/cm to 150 kV/cm. The states were calculated using an eight-band $\mathbf{k} \cdot \mathbf{p}$ strain-dependent Hamiltonian²⁷ in the envelope approximation using finite differences on a real-space grid²⁸. This method has been used previously to calculate g tensors of electrons in both quantum dots^{8,12} and bound to donors⁹.

The large electric fields considered here would ionize an electron in these dots⁸, however holes remain confined due to their large effective mass. Whereas bulk holes have $J = 3/2$, in a self-assembled dot the geometric asymmetry and strain break the four-fold degeneracy, resulting in doubly degenerate levels that are mixtures of heavy and light holes. Although the lowest hole state is a doublet (mostly heavy hole, with small light hole components), none of the eight envelope functions is identically zero and therefore the doublet energies will be split by a magnetic field pointing in any direction. The hole g tensor component for a magnetic field B applied in a direction $\hat{\alpha}$ was found by calculating the splitting ΔE to obtain $|g_{\hat{\alpha}}| = \Delta E / 3\mu_B B$, where μ_B is the Bohr magneton. We follow the convention for holes of including a factor of 3 in the denominator (reflecting $m_J = \pm 3/2$) even though the states form a two-level system and are not pure heavy hole. The sign of $g_{\hat{\alpha}}$ was determined from the spin orientation of the ground state wave function, with $g_{\hat{\alpha}} > 0$ for the spin pointing antiparallel to \mathbf{B} . Since the lens-shaped

dots were elongated along the $[110]$ direction the principal axes were $[110]$, $[\bar{1}\bar{1}0]$, and $[001]$. Spin splittings were computed for \mathbf{B} along each of these symmetry directions to obtain each $g_{\hat{\alpha}}$.

B. g tensor dependence on electric field

Earlier investigations of electron spin manipulation using an electrically controlled g tensor identified the importance of having a g tensor component that changes sign with electric field⁸. If the sign of one $g_{\hat{\alpha}}$ can be changed with an applied electric field, then it is possible to precess the spin to point in an arbitrary direction by only changing the electric field (in the presence of a suitably oriented static magnetic field). We have identified a dot geometry having the desired sign change, with a height of 6.2 nm, a 21.6 nm base along the minor axis ($[110]$ direction) and 32.8 nm along the major axis ($[\bar{1}\bar{1}0]$ direction). The g tensor of this dot as a function of electric field is plotted in Fig. 1. The most interest-

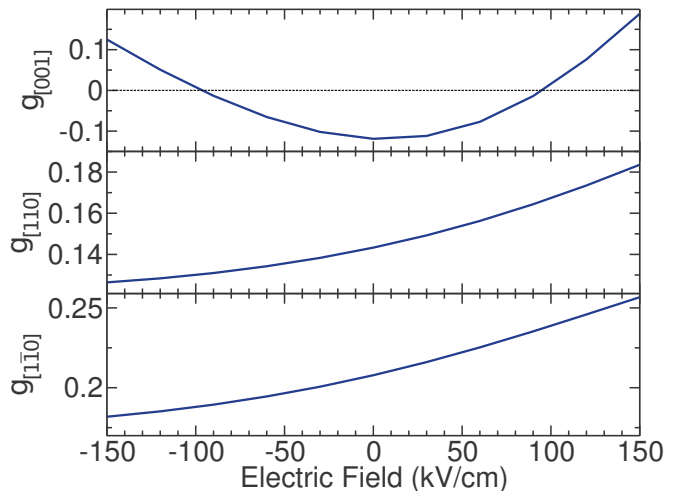


FIG. 1: (color online) g tensor of the uppermost valence state in an $\text{In}_{0.5}\text{Ga}_{0.5}\text{As}/\text{GaAs}$ quantum dot as a function of electric field at 0K. The dot has a height of 6.2nm, footprint length along the $[110]$ direction of 21.6nm, and footprint length along the $[\bar{1}\bar{1}0]$ direction of 32.8nm. Of particular note is the sign change of the g tensor component along the $[001]$ direction.

ing component is $g_{[001]}$ which is concave upward with a minimum value $g_{[001]} \approx -0.1$ at $E \approx 0$ and $g_{[001]} \approx 0$ at $E \approx \pm 100$ kV/cm. Therefore, there are two possible electric field values around which nonresonant spin manipulation may be done over the entire Bloch sphere as described in Ref. 8. This double sign change limits the range over which such spin manipulation may be performed, as the limited range of $|g_{[001]}|$ constrains the effective spin precession frequency. However, the field ranges involved correspond roughly to the breakdown fields in GaAs, so larger electric fields are unlikely to be practical.

In Fig. 1 we see a significant nonlinearity in $g_{[001]}$, whereas the nonlinearities in $g_{[110]}$ and $g_{[1\bar{1}0]}$ are much smaller. These nonlinearities were parameterized by fitting a second order polynomial in E to the g tensor components, resulting in the coefficients given in Table I.

$\hat{\alpha}$	[001]	[110]	[1 $\bar{1}$ 0]
c	-0.115	0.143	0.208
b (cm/kV)	1.15×10^{-4}	1.88×10^{-4}	2.52×10^{-4}
a (cm ² /kV ²)	1.23×10^{-5}	5.20×10^{-7}	5.08×10^{-7}

TABLE I: Coefficients of a fit of the g tensor in Fig. 1 to $g_{\hat{\alpha}} = a_{\hat{\alpha}}E^2 + b_{\hat{\alpha}}E + c_{\hat{\alpha}}$, where E is the electric field component along [001].

III. SPIN MANIPULATION USING g TENSOR MODULATION WITH AN ELECTRIC FIELD

A. Nonresonant hole spin manipulation with an electric field

We first consider nonresonant spin precession using the technique developed in Ref. 8. By applying two different electric fields (E_1 and E_2) along the growth direction, precession around two orthogonal axes may be obtained. This approach requires the g tensor component along one symmetry direction (here [001]) to change sign as a function of E . For a given E_1 and E_2 such that $g_{[001]}(E_1)g_{[001]}(E_2) < 0$, the magnetic field direction required to obtain orthogonal spin precession axes $\mathbf{\Omega} = \tilde{\mathbf{g}} \cdot \mathbf{B}$ is determined by the condition

$$(\tilde{\mathbf{g}}(E_1) \cdot \mathbf{B}) \cdot (\tilde{\mathbf{g}}(E_2) \cdot \mathbf{B}) = 0. \quad (1)$$

The optimal solution is determined by maximizing $|\Omega(E_1)|$ subject to $|\Omega(E_1)| = |\Omega(E_2)|$ and $|E_{1,2}| < 150$ kV/cm (to avoid breakdown). For the dot geometry corresponding to Fig. 1 we obtain $E_1 = -150$ kV/cm and $E_2 = 3.1$ kV/cm. The optimal magnetic field angle (0.24π) is nearly $\pi/4$, measured from the [001] axis towards the [110] axis. For a magnetic field of 5 Tesla the time for the spin to precess by π is 18 ps.

B. Resonant hole spin manipulation with an electric field

For a g tensor that depends linearly on the electric field, resonances occur when the oscillation frequency of the electric field match the Larmor frequency of the spin in the static magnetic field. However, when the g tensor depends nonlinearly on the electric field, resonance may occur at subharmonics of the Larmor frequency⁹. The strongly nonlinear behavior of the g tensor for holes,

evident in Fig. 1 and parametrized in Table I, produces such subharmonic resonances. For an applied field $E(t) = E_{dc} + E_{ac} \sin(\omega t)$, the response amplitudes are

$$\begin{aligned} \Omega_{\hat{\alpha}}(t) &= B_{\hat{\alpha}} (a_{\hat{\alpha}}E^2(t) \sin^2(\omega t) + b_{\hat{\alpha}}E(t) \sin(\omega t) + c_{\hat{\alpha}}) \\ &= \Omega_{0,\hat{\alpha}} + \Omega_{1,\hat{\alpha}} \sin(\omega t) - \Omega_{2,\hat{\alpha}} \cos(2\omega t), \end{aligned} \quad (2)$$

where

$$\Omega_{0,\hat{\alpha}} = B_{\hat{\alpha}} ([E_{dc}^2 + E_{ac}^2/2] a_{\hat{\alpha}} + E_{dc}b_{\hat{\alpha}} + c_{\hat{\alpha}}), \quad (3)$$

$$\Omega_{1,\hat{\alpha}} = B_{\hat{\alpha}} E_{ac} (2E_{dc}a_{\hat{\alpha}} + b_{\hat{\alpha}}), \quad (4)$$

$$\Omega_{2,\hat{\alpha}} = B_{\hat{\alpha}} E_{ac}^2 a_{\hat{\alpha}}/2. \quad (5)$$

Ω_1 is the response at the fundamental $\omega = \omega_0$ and Ω_2 is the response at the subharmonic $\omega = \omega_0/2$, where ω_0 is the Larmor precession frequency. Higher order polynomial dependences (*e.g.* cubic or quartic) of the g tensor on the electric field will result in resonances at additional subharmonics of the Larmor frequency. Higher-order effects, including the counter-rotating components of the oscillating transverse component of the spin precession vector, will bring additional shifts of the lower-order resonances and bring in resonances at other multiples of the Larmor frequency.²⁹

The precession rates for the subharmonic and fundamental resonances were calculated to first order for the full range of magnetic field angles, and for electric field amplitudes less than the breakdown field ~ 200 kV/cm. The Rabi frequency associated with the fundamental resonance was found for an electric field oscillating about $E_{dc} = 0$ as a function of applied magnetic field direction, ϕ , in the [001]-[1 $\bar{1}$ 0] plane and as a function of electric field amplitude E_{ac} . For any given value of E , an optimal magnetic field direction was found, corresponding to the largest Rabi frequency. As a function of E , the optimum magnetic field direction increases monotonically. At 200 kV/cm the largest Rabi frequency for a magnetic field of 10 Tesla is 18 GHz at a optimal magnetic field angle of 1.2 radians from the [001] axis. The time required for the spin to precess by π in this configuration is ~ 28 ps.

The Rabi frequency for the subharmonic resonance was also found for an electric field oscillating about $E_{dc} = 0$ as a function of applied magnetic field direction ϕ and electric field amplitude E . As with the fundamental resonance, a general trend of faster spin manipulation times at higher E was found through $E = 200$ kV/cm, peaking at an angle of 1.2 radians from the [001] axis. At $B = 10T$ the peak Rabi frequency was 39 GHz, corresponding to a minimal time for the spin to precess by π of 13 ps.

IV. CONCLUDING REMARKS

We have examined theoretically the hole g tensors in $\text{In}_{0.5}\text{Ga}_{0.5}\text{As}/\text{GaAs}$ quantum dots for possible application to hole spin manipulation. A structure was pro-

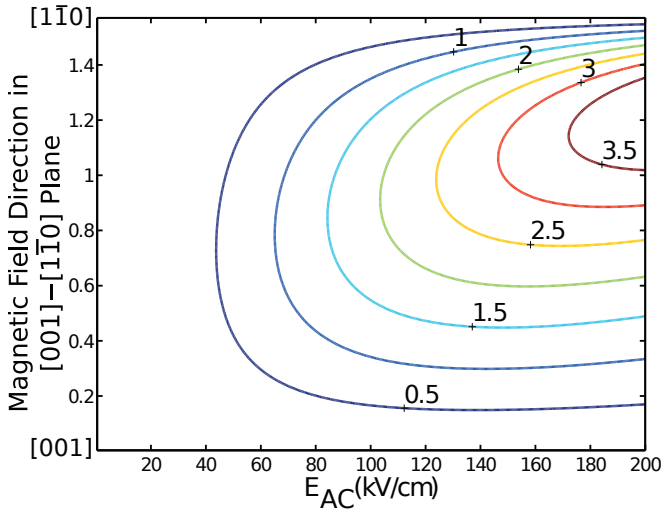


FIG. 2: (color online) Contour plot of Rabi frequency (in GHz) at quadratic resonance as a function of AC electric field amplitude E_{ac} and magnetic field angle, for $B=1\text{T}$.

posed for which one g tensor component sign changed as a function of electric field applied along the $[001]$ growth direction. The nonlinear g tensor dependence on applied electric field causes a subharmonic resonance to appear at $\omega = \omega_0/2$, with higher-order dependencies generating further subharmonics. The g tensor for this structure was used to calculate resonant and nonresonant spin manipulation frequencies. For a magnetic field of 10 Tesla, the resonant spin manipulation method had an optimal spin manipulation time (precession by π) of 28 ps at the fundamental resonance, or 13 ps at the subharmonic resonance. The nonresonant spin manipulation frequency for a 5 Tesla field was 18 ps. Using the experimental values of T_2 from²⁵, approximately 10^4 operations would be possible during a T_2 time.

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